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Optimal Control of Hot Oil Preparation for Natural Gas Fractionation

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***Abstract*—This paper introduces optimal control into the process of oil heating for natural gas fractionation. There are two sources of thermal energy for heating the oil, one of which is a free resource because it uses waste heat from the cogeneration process. With anticipation of the thermal loads related to gas fractionation one day in advance, it is possible to employ supervisory predictive control for minimisation of operative costs. The underlying optimization problem is solved using linear programming through which optimal references for the low-level circuits are obtained. The performed simulation tests show that the desired control effect is achieved, that is, a constant hot oil supply with the required amount and temperature while adhering to all technical limitations and with minimal operating costs.**

I. INTRODUCTION

The energy systems worldwide are currently facing the issues of decarbonisation and prices volatility. The most pronounced recent increase of energy prices was in the price of natural gas. Natural gas prices quadrupled in Europe, trippled in Asia, and doubled in the US in 2021 [1]. Both industrialized and developing countries have to rethink their energy transition plans in light of today's circumstances. It is required not only to limit the environmental impact of fossil fuel production and consumption but also to ensure energy security, reliability, access, affordability, and sustainability.

Limiting the consumption of natural resources and reducing the emission of waste gases from the process with the aim of preserving the environment and meeting all strict legal regulations, as well as saving raw materials and energy, represent increasingly important requirements of the industry. It is necessary to apply technologies that, in addition to fulfilling the previously mentioned requirements, will increase efficiency and simplify operations. Therefore, the aim is to find an optimal way of managing the underlying industrial processes. The computation of optimal schedules and their application directly to the process or for operators decision support has not been widely applied in the past. The primary reason for this is the complex nature of industrial processes, which frequently results in mathematical programming problems that are hard to solve in real-time.

Paper [2] presents an approach to scheduling based on the use of hybrid systems and model predictive control (MPC) [3]. It is based on two MPC instances working in cascade and thus forming outer loop MPC and inner loop MPC. The plant economic goals are the basis for the computation of reference schedules by the outer loop MPC. The objective of the inner loop is to respond to plan deviations in a way that allows the plant to adjust to changing conditions while maintaining as much of the original schedule as possible. It is often that the inner loops are executed by simple PID controllers, which are responsible for keeping the plant at the setpoints set by the outer loop MPCs. MPC is more and more used for control and optimization of industrial processes, especially to gain efficiency in transients involving several control variables. For example, during the startup and shutdown of large generating units, MPC algorithms determine optimal setpoints for temperatures, pressures, and fuel feed rates [4].

Thermal energy storages (TESs) are very important for flexibility in heating systems. Authors in [5] present an MPC procedure to schedule a heat source within a heating system containing a TES. Using sequential/successive mixed-integer linear programming (SMILP), the problem of day-ahead optimal scheduling of a heat source connected to a TES is developed in a MPC framework. The results showed that this is far better than the PI controller in terms of energy efficiency and meeting the given constraints. Reference [6] presents the comparative performance of MPC and PID controller in controlling temperature on a Small Scale Industry Steam Distillation Pilot Plant, where MPC controller was capable to meet the required transient quality providing reduced sampling time and overshoot when compared with a PID controller.

Reusing waste heat from oil and gas burning processes is one of the most important methods in energy conservation and carbon emission reduction research. In [7] authors examine recovery and cascade utilization of waste heat for low-grade flue gas in industrial field. Integrated heating, cooling, and electricity generation plants composed of a gas turbine, a natural gas liquids (NGL) refinery, and an absorption chiller

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were proposed as a hybrid system in [8]. Thermo-economic analysis showed that the proposed system is very economical even at low electricity prices.

In this paper we introduce optimal control to an industrial process of oil heating for natural gas fractionation [9]. In the natural gas fractionation process, it is necessary to ensure the flow of a sufficient amount of heating medium at the appropriate temperature for its smooth development, where maintaining a constant oil temperature in a heat exchanger is desirable for consistent and efficient heating of the natural gas in the fractionation process. In this case, the hot oil heating medium is circulated through the associated tank and is heated partly by waste heat from the cogeneration process and partly by the use of additional gas heaters. By using the oil tank and predictive control of heating the oil, it is possible to ensure optimal utilization of waste heat and to minimize additional consumption of natural gas on the heater. We show how the supervisory control layer can be structured in order to maintain simplicity in computation of optimal references supplied to low-level controllers even for longer time horizons.

The paper is organized as follows: Section II describes the system, and its mathematical model is given in Section III. The optimization problem for supervisory MPC for oil heating is formulated in Section IV, and the results of its application in junction with classical multi-variable low-level controllers are presented in Section V. Also, the comparison with classic non-predictive control method is given here.

II. SYSTEM DESCRIPTION

Natural gas is a mixture of lower hydrocarbons, inorganic compounds and gases. The main ingredient is methane with a share of more than 70%. In addition to methane, lower proportions of ethane, propane and other higher alkanes can be found in the composition of natural gas. Inorganic ingredients are represented by carbon dioxide, nitrogen and hydrogen sulphide. Before actual use and distribution, natural gas must be pre-processed. Acid gases (CO_2, H_2S, COS) and mercury are removed first. Higher hydrocarbons are also separated and can serve as separate raw materials. The process of separating individual hydrocarbons from natural gas is called degasolination. There are several different procedures, and one of them is separation by fractionation of the liquefied mixture at low temperatures.

The fractionation process is carried out so that the natural gas is first cooled to a temperature of -100 °C by special procedures and it becomes a liquid mixture, commonly called natural gas liquids (NGLs). By the distillation process at reduced pressure, with an increase in temperature, methane, ethane, propane, butane, and finally primary gasoline are separated in the columns. The lightest (lowest boiling point) NGL component boils off first and separates as the NGL stream heats up. The process is repeated until the NGLs have been separated into their individual components [9].

In this case, hot oil is used as a medium for heating the bottom of the fractionation columns through the heat exchanger. This part of the process plant represents a separate technological unit whose goal is to ensure the supply of a sufficient amount of oil at the appropriate temperature so that the fractionation process can proceed smoothly. The optimal control of the specific process of heating hot oil for natural gas fractionation is presented in the sequel.

The oil circulates through the hot oil tank and after leaving the tank it is heated partly by waste heat from the cogeneration process, and partly by using additional gas heaters. Fig. 1 represents a simplified scheme of the hot oil system. As part of the industrial plant, there is a combined heat-power (CHP) generation unit for the production of electricity for the needs of the plant, powered by a gas turbine. The by-product at the exit from gas turbines are flue gases. Their heat is used to heat the oil through the hot oil utilization boiler - heat recovery unit. With the help of a control valve (diverter) and flue ducts, the flue gases can be directed into the exchange section of the heat recovery unit or directly through the chimney into the atmosphere, which enables regulation of the oil temperature by the amount of flue gases that pass through the exchange section. Thus, this method of heating uses waste heat and is therefore the primary method of heating the oil. When necessary, the oil is also heated using an additional gas heater. The gas supply is controlled by the regulating valve. In order to achieve optimal control of the hot oil preparation process, it is necessary to minimize gas consumption on the heater. This can be achieved by using a hot oil tank in such a way that, knowing the expected consumption profile and waste heat availability, the mode of operation is planned a day in advance, i.e. the operating profile of the heat recovery unit and heater at any moment and thus the usage of the tank's thermal capacity is optimally exploited.

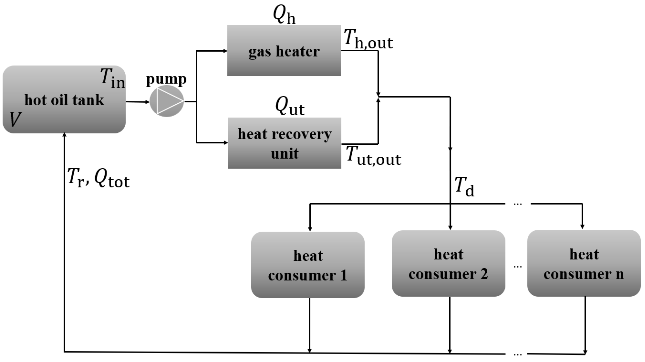

Fig. 1. A simplified scheme of the hot oil system

III. MATHEMATICAL MODEL

The main requirement of this process is the desired hot oil inlet temperature T_d. It is the result of heating in the heater and heat recovery sections:

$$T_d = \frac{Q_h T_{h,out} + Q_{ut} T_{ut,out}}{Q_h + Q_{ut}}, \tag{1}$$

where the notation is as follows:

- Q_{h}: oil flow through the gas heater in kg/s, and it is equal $Q_{\mathrm{h}} = k_{\mathrm{v,h}} \frac{x_{\mathrm{v,h}}}{100\%} \sqrt{\Delta P}$, where $k_{\mathrm{v,h}}$ is the constant of the gas heater regulation valve, $x_{\mathrm{v,h}}$ is percentage of the valve opening, and ΔP is pressure difference ensured by the hot oil circulation pump,
- Q_{ut}: oil flow through the utilization unit in kg/s, and it is equal $Q_{\mathrm{ut}} = k_{\mathrm{v,ut}} \frac{x_{\mathrm{v,ut}}}{100\%} \sqrt{\Delta P}$,
- $T_{\mathrm{h,out}}$: oil temperature after heating in the heater section in °C,
- $T_{\mathrm{ut,out}}$: oil temperature after heating in the utilization section in °C.

The powers of the heater and utilization unit are:

$$P_{\mathrm{h}} = C_{\mathrm{p}} Q_{\mathrm{h}} (T_{\mathrm{h,out}} - T_{\mathrm{in}}), \tag{2}$$

$$P_{\mathrm{ut}} = C_{\mathrm{p}} Q_{\mathrm{ut}} (T_{\mathrm{ut,out}} - T_{\mathrm{in}}), \tag{3}$$

where the notation is as follows:

- P_{h}: the power of the heater in W,
- P_{ut}: the power of the utilization unit in W,
- C_{p}: oil specific heat capacity in J/kg°C,
- T_{in}: oil temperature in the hot oil tank in °C.

As the diverter valve regulates which part of the flue gas heat is utilized by the utilization unit and which part is released into the atmosphere, the quantity d_{ut} is also introduced, which describes the relationship between the operating power and the maximum power that can be provided ($d_{\mathrm{ut}} = P_{\mathrm{ut}}/P_{\mathrm{ut,max}}$). Also, the same quantity is introduced for the heater, since its power is controlled by a valve that regulates the flow of gas to it, i.e. it regulates what portion of the maximum possible power the heater will work with ($d_{\mathrm{h}} = P_{\mathrm{h}}/P_{\mathrm{h,max}}$). Therefore, the oil temperatures at the outlets of each section are equal to:

$$T_{\mathrm{h,out}} = T_{\mathrm{in}} + \frac{d_{\mathrm{h}} P_{\mathrm{h,max}}}{C_{\mathrm{p}} Q_{\mathrm{h}}}, \tag{4}$$

$$T_{\mathrm{ut,out}} = T_{\mathrm{in}} + \frac{d_{\mathrm{ut}} P_{\mathrm{ut,max}}}{C_{\mathrm{p}} Q_{\mathrm{ut}}}. \tag{5}$$

So, by including (4) and (5) in (1):

$$T_{\mathrm{d}} = T_{\mathrm{in}} + \frac{P_{\mathrm{ut}} + P_{\mathrm{h}}}{C_{\mathrm{p}} (Q_{\mathrm{ut}} + Q_{\mathrm{h}})}. \tag{6}$$

Heat sinks represent a heat load. They are observed as a disturbance in the hot oil temperature control system which is known to us one day in advance and is defined as consumption power P_{con}:

$$P_{\mathrm{con}} = C_{\mathrm{p}} Q_{\mathrm{tot}} (T_{\mathrm{d}} - T_{\mathrm{r}}), \tag{7}$$

where the notation is as follow:

- Q_{tot}: total oil flow, in kg/s, and it is equal to $Q_{\mathrm{tot}} = \frac{P_{\mathrm{h}} + P_{\mathrm{ut}}}{C_{\mathrm{p}} (T_{\mathrm{d}} - T_{\mathrm{in}})}$,
- T_{r}: oil temperature at the return to the hot oil tank, in °C.

The heat losses of the tank P_{loss} are also included:

$$P_{\mathrm{loss}} = \frac{T_{\mathrm{in}} - T_{\mathrm{a}}}{R}, \tag{8}$$

where the notation is as follows:

- T_{a}: air temperature in °C, will be taken as constant during the optimization period,
- R: thermal resistance of the tank in °C/W, and it is equal to $R = \frac{l}{\lambda A_{\mathrm{tank}}}$, where l is insulator thickness, λ is thermal conductivity and A_{tank} is the surface area of the tank (a cylinder-shaped tank with $h_{\mathrm{tank}} = 4 r_{\mathrm{tank}}$ is considered here).

Now the dynamics of the process, i.e. the change of the stored thermal energy in the hot oil tank in a unit of time can be written down. It is equal to the difference between incoming and outgoing heat flow:

$$C \frac{\mathrm{d}T_{\mathrm{in}}}{\mathrm{d}t} = C_{\mathrm{p}} Q_{\mathrm{tot}} T_{\mathrm{r}} - C_{\mathrm{p}} Q_{\mathrm{tot}} T_{\mathrm{in}} - \frac{T_{\mathrm{in}} - T_{\mathrm{a}}}{R}, \tag{9}$$

where C is the thermal capacity of the tank and it is equal to $C = \rho_{\mathrm{oil}} V C_{\mathrm{p}}$. By including the previous equations in the initial one, the fundamental dynamic equation of the oil heating process is obtained. It shows how the state of the process (oil temperature in the tank that enables optimization) changes depending on the inputs to the process that are to be set by optimal control (power of the heater and heat recovery unit) and on disturbance - consumption power:

$$\frac{\mathrm{d}T_{\mathrm{in}}}{\mathrm{d}t} = \frac{1}{\rho V C_{\mathrm{p}}} (P_{\mathrm{h}} + P_{\mathrm{ut}} - P_{\mathrm{con}} - P_{\mathrm{loss}}). \tag{10}$$

The discretized model with sampling time T_{s} is:

$$\begin{aligned} T_{\mathrm{in}}(k+1) = (1 - \frac{T_{\mathrm{s}}}{\rho V C_{\mathrm{p}} R}) T_{\mathrm{in}}(k) + \\ + \frac{T_{\mathrm{s}}}{\rho V C_{\mathrm{p}}} (P_{\mathrm{h}}(k) + P_{\mathrm{ut}}(k) - P_{\mathrm{con}}(k) + \frac{T_{\mathrm{a}}}{R}), \end{aligned} \tag{11}$$

where k is the sampling time index, $P(k)$ are medium powers on the interval $[kT_{\mathrm{s}}, (k+1)T_{\mathrm{s}})$ and $T_{\mathrm{in}}(k)$ is temperature at kT_{s}.

The model can be written in the standard form as:

$$x(k+1) = A x(k) + B_{\mathrm{u}} u(k) + B_{\mathrm{d}} d(k), \tag{12}$$

where the system state vector is $x(k) = T_{\mathrm{in}}(k)$, the system input vector is $u(k) = [P_{\mathrm{h}}(k), P_{\mathrm{ut}}(k)]^T$ and the disturbance is $d(k) = [P_{\mathrm{con}}(k), T_{\mathrm{a}}/R]^T$.

IV. SYSTEM OPTIMAL SCHEDULING

A. Cost Function and Constraints

In the sequel the aim is to find the optimal schedule of the hot oil temperature control system for one full day ahead, with the heat consumption P_{con} predicted for that following day. Moreover, in order to find the most cost-optimal solution possible to serve the required needs, the best initial state will be sought as well with included requirement for repeatability of such operation day-after-day with presumed repeating disturbance sequence P_{con}. The cost function and constraints for such an optimisation will be written in a form of a linear program (LP):

$$\begin{aligned} &\min f^T z \\ \text{s.t.}\quad &A_{\text{ineq}} z \le b_{\text{ineq}}, \\ &A_{\text{eq}} z = b_{\text{eq}}, \\ &lb \le z \le ub, \end{aligned} \tag{13}$$

where z is the vector of optimisation variables and it contains: $T_{\text{in}}(0)$, $P_{\text{h}}(k)$ and $P_{\text{ut}}(k)$. Vector f is the vector of cost function coefficients, A_{ineq} and b_{ineq} are matrix and vector of inequality constraints, and A_{eq} and b_{eq} are matrix and vector of equality constraints, lb and ub are lower and upper bounds of z.

As said before, the goal is to minimize energy consumption on the gas heater, so the cost function is defined as:

$$\min \sum_{k=0}^{N-1} P_{\text{h}}(k) T_{\text{s}}, \tag{14}$$

where the number of sampling instants N considered forms the prediction horizon of 24 hours ahead with $T_{\text{s}} = 15$ min as sampling time.

The constraints are characterized as follows.

- The powers of the utilization unit and the heater are limited by a maximum value, where the maximum power that can be provided by the heater is constant and equals $P_{\text{h,max}}$, while the maximum power of the utilization unit at any moment depends on the production of electrical energy at that moment:

$$\begin{aligned} 0 &\le P_{\text{h}}(k) \le P_{\text{h,max}}, \\ 0 &\le P_{\text{ut}}(k) \le P_{\text{ut,max}}(k). \end{aligned} \tag{15}$$

- The constraint for maximum total power at every moment derives from the existence of the maximum total flow that is realized when both valves, which control flow through section of heater and section of utilization unit, are completely (100%) open:

$$P_{\text{h}}(k) + P_{\text{ut}}(k) \le (k_{\text{v,ut}} + k_{\text{v,h}}) \sqrt{\Delta P} C_{\text{p}} (T_{\text{d}} - T_{\text{in}}(k)), \tag{16}$$

- Next constraint is the lower limit of thermal oil operating temperature $T_{\text{r}}(k) \ge T_{\text{r,min}}$:

$$T_{\text{r}}(k) = T_{\text{d}} - \frac{P_{\text{con}}(k)}{C_{\text{p}} Q_{\text{tot}}(k)} \ge T_{\text{r,min}} \tag{17}$$

By including equation for total oil flow $Q_{\text{tot}} = \dfrac{P_{\text{h}} + P_{\text{ut}}}{C_{\text{p}}(T_{\text{d}} - T_{\text{in}})}$ this becomes the constraint for minimal total power at every moment:

$$P_{\text{h}}(k) + P_{\text{ut}}(k) \ge \frac{T_{\text{d}} - T_{\text{in}}(k)}{T_{\text{d}} - T_{\text{r,min}}} P_{\text{con}}(k). \tag{18}$$

- Since the temperature in the hot oil tank changes uniformly in time interval $[kT_{\text{s}}, (k+1)T_{\text{s}})$ from $T_{\text{in}}(k)$ to $T_{\text{in}}(k+1)$, one needs to ensure that the constraints on powers (16) and (18) are ensured for both extreme values of temperature on the interval, so both for $T_{\text{in}}(k)$ and for $T_{\text{in}}(k+1)$, and thus they are also written with temperature $T_{\text{in}}(k+1)$:

$$P_{\text{h}}(k) + P_{\text{ut}}(k) \le (k_{\text{v,ut}} + k_{\text{v,h}}) \sqrt{\Delta P} C_{\text{p}} (T_{\text{d}} - T_{\text{in}}(k+1)), \tag{19}$$

$$P_{\text{h}}(k) + P_{\text{ut}}(k) \ge \frac{T_{\text{d}} - T_{\text{in}}(k+1)}{T_{\text{d}} - T_{\text{r,min}}} P_{\text{con}}(k). \tag{20}$$

- Lower limit of oil temperature in the tank is the minimum thermal oil operating temperature $T_{\text{in,min}} = T_{\text{r,min}}$, and upper limit is $T_{\text{in,max}}$:

$$T_{\text{in,min}} \le T_{\text{in}}(k) \le T_{\text{in,max}}. \tag{21}$$

- The last constraint is here to ensure repeatability and it is the only equality constraint:

$$T_{\text{in}}(0) = T_{\text{in}}(N). \tag{22}$$

The listed constraints and the cost function form the following optimization problem:

$$\begin{aligned} &\min \sum_{k=0}^{N-1} u_1(k) T_{\text{s}}, \\ &\text{s.t.} \\ &\quad x(k+1) = Ax(k) + B_{\text{u}} u(k) + B_{\text{d}} d(k), \\ &\quad 0 \le u_1(k) \le P_{\text{h,max}}, \\ &\quad 0 \le u_2(k) \le P_{\text{ut,max}}(k), \\ &\quad \frac{T_{\text{d}} - x(k)}{T_{\text{d}} - T_{\text{r,min}}} d_1(k) \le u_1(k) + u_2(k) \le c(T_{\text{d}} - x(k)), \\ &\quad \frac{T_{\text{d}} - x(k+1)}{T_{\text{d}} - T_{\text{r,min}}} d_1(k) \le u_1(k) + u_2(k) \le c(T_{\text{d}} - x(k+1)) \\ &\quad T_{\text{in,min}} \le x(k) \le T_{\text{in,max}}, \\ &\quad x(0) = x(N). \end{aligned} \tag{23}$$

where c is constant, $c = (k_{\text{v,ut}} + k_{\text{v,h}}) \sqrt{\Delta P} C_{\text{p}}$.

B. Control algorithm

By solving the LP (23) the optimal daily schedule for the powers of the gas heater $P_{\text{h}}^*(k)$ and the utilization unit $P_{\text{ut}}^*(k)$, and the optimal initial state $T_{\text{in}}^*(0)$, are obtained for the whole 24-hour period. From these values the optimal total flow in every time sample is derived:

$$Q_{\text{tot}}^*(k) = \frac{P_{\text{h}}^*(k) + P_{\text{ut}}^*(k)}{C_{\text{p}}(T_{\text{d}} - T_{\text{in}}(k))}, \tag{24}$$

and thus optimal references of flow and temperature of oil through both sections (heater section and utilization unit section) for the control algorithm are obtained, where the equations (1) and (6) must hold at all times. These references are calculated during system operation, as they take into account the actual, currently established T_{in}, which enhances the system's robustness. Control law is designed to achieve the desired temperature by regulation of each heating section to T_{d}. Sometimes that is not possible due to the constraints of the corresponding flows, so the resulting T_{d} is obtained by combining higher and lower temperatures in the corresponding sections. The algorithm for keeping the optimal flows and the optimal outlet temperatures is given in Algorithm 1.

Algorithm 1 Control algorithm

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Input: $P^*_{\mathrm{h}}(k)$, $P^*_{\mathrm{ut}}(k)$, $T_{\mathrm{in}}(k)$
Output: $Q^*_{\mathrm{h}}(k)$, $Q^*_{\mathrm{ut}}(k)$, $T^*_{\mathrm{h,out}}(k)$, $T^*_{\mathrm{ut,out}}(k)$
If $\frac{P^*_{\mathrm{ut}}(k)}{C_{\mathrm{p}}(T_{\mathrm{d}}-T_{\mathrm{in}}(k))} \geq k_{\mathrm{v,ut}}\sqrt{\Delta P}$
        $Q^*_{\mathrm{ut}}(k) = k_{\mathrm{v,ut}}\sqrt{\Delta P}$
        $Q^*_{\mathrm{h}}(k) = Q^*_{\mathrm{tot}}(k) - Q^*_{\mathrm{ut}}(k)$
Else
    If $\frac{P^*_{\mathrm{h}}(k)}{C_{\mathrm{p}}(T_{\mathrm{d}}-T_{\mathrm{in}}(k))} \geq k_{\mathrm{v,ut}}\sqrt{\Delta P}$
        $Q^*_{\mathrm{h}}(k) = k_{\mathrm{v,ut}}\sqrt{\Delta P}$
        $Q^*_{\mathrm{ut}}(k) = Q^*_{\mathrm{tot}}(k) - Q^*_{\mathrm{h}}(k)$
    Else
        $Q^*_{\mathrm{h}}(k) = \frac{P^*_{\mathrm{h}}(k)}{C_{\mathrm{p}}(T_{\mathrm{d}}-T_{\mathrm{in}}(k))}$
        $Q^*_{\mathrm{ut}}(k) = \frac{P^*_{\mathrm{ut}}(k)}{C_{\mathrm{p}}(T_{\mathrm{d}}-T_{\mathrm{in}}(k))}$
    End if
End if
$T^*_{\mathrm{h,out}}(k) = T_{\mathrm{in}}(k) + \frac{P^*_{\mathrm{h}}(k)}{C_{\mathrm{p}}Q^*_{\mathrm{h}}(k)}$
$T^*_{\mathrm{ut,out}}(k) = T_{\mathrm{in}}(k) + \frac{P^*_{\mathrm{ut}}(k)}{C_{\mathrm{p}}Q^*_{\mathrm{ut}}(k)}$
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In Fig. 2 the control circuits for the utilization section or the heater section are shown, for closed-loop maintenance of flows and output temperatures in accordance with the cost-optimal computed day-ahead operation schedule. These two sections are separate multiple-input-multiple-output (MIMO) control circuits, each with two inputs ($u_1 = x_{\mathrm{v,ut/h}}$, $u_2 = d_{\mathrm{ut/h}}$) and two outputs ($y_1 = Q_{\mathrm{ut/h}}$, $y_2 = T_{\mathrm{ut/h,out}}$), where u_1 acts on y_1 and y_2, while u_2 acts only on y_2 - in other words these are unilaterally coupled MIMO processes. Here notation ${}^*{}_{\mathrm{ut/h}}$ denotes that the variable * belongs to the MIMO circuit of the utilization unit when 'ut' is used and to the heater unit when 'h' is used. The two circuits are of exactly the same structure, so with this notation a more compact description is achieved. Controller-based decoupling in these MIMO control-circuits is ensured with the decoupling element $K_{\mathrm{ut/h}}$, and then, $R_{1\mathrm{ut/h}}$ and $R_{2\mathrm{ut/h}}$ are designed as classic PI controllers whose parameters are changing based on operating point i.e. the reference calculated by optimal day-ahead predictive scheduling for every 15 minutes time period.

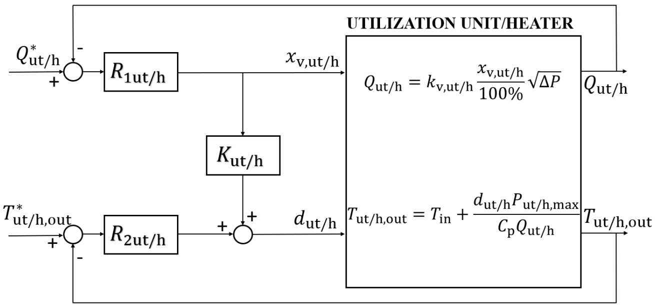

Fig. 2. Control circuits outlook. The same outlook holds for both utilization unit and gas heater (notice the '/' sign used to outline both of them with a single scheme).

V. RESULTS

The derived procedure of optimal day-ahead scheduling of the hot oil temperature control system is tested for a concrete system set-up described with parameters in Table I.

TABLE I
SYSTEM PARAMETERS

Parameter	Value
T_{d} - normal operating temperature of the hot oil inlet	220 °C
$T_{\mathrm{in,max}}$ - maximum oil temperature at the entrance to the pump	160 °C
$T_{\mathrm{min}} = T_{\mathrm{r,min}} = T_{\mathrm{in,min}}$ - minimum thermal oil operating temperature	120 °C
T_{a} - air temperature	20 °C
$Q_{\mathrm{tot,max}}$ - maximum total flow of the oil	48.6 kg/s
$P_{\mathrm{h,max}}$ - maximum power of the heater	4 MW
ρ - oil density	729 kg/m^3
V - tank volume	74 m^3
C_p - specfic heat capacity of the oil	2.61 kJ/kg°C
ΔP - difference in pressure level at the inlet and outlet of a valve	$3 \cdot 10^5$ Pa
l - insulator thickness	0.1 m
λ - thermal conductivity	0.033 W/mK

The LP optimization problem is solved in Matlab [10] and solver used for that is IBM ILOG CPLEX [11]. Thus optimal references of flow and temperature of oil through both sections (heater section and utilization unit section) for the control algorithm were obtained. The control system is designed and simulated in Matlab/Simulink environment. Fig. 3(a) shows how maximum power of utilization unit and thermal loads are predicted to change during the day, the predicted consumption power, and the optimization result in the form of optimal power profiles for the utilization unit and the heater. Control law is designed to achieve the desired temperature of 220 °C with minimal costs by making maximum use of utilization unit, and only what it cannot provide is provided by the heater. For this reason, the temperature in the utilization unit section rises above 220 °C, and in the heater section it is kept below that level so that heater is engaged as little as possible, and then the resulting 220 °C is obtained by combining higher and lower temperatures in the corresponding sections. Fig. 3(c) shows oil temperature at section outlets and the final oil temperature that goes to the consumers during 24 hours. The desired result is obtained, that is, a constant supply to the consumers with the required amount of hot oil at a temperature of 220 °C while observing all technical restrictions and with minimal operating costs. In Fig. 3(e) it can be seen how oil temperature in the tank changes during the day compared to the calculated optimal one. The deviations that occur at specific moments are the result of finite speed of the designed local closed-loop controls. Finally, the total amount of energy consumed by the heater during the day is 99.085 TJ.

Fig.3 also shows the behavior of the system when controlled by a classic non-predictive method such as hysteresis. The system can operate either with the aim of satisfying the final temperature to the consumers T_{d} or with the aim of keeping the oil temperature in the tank T_{in} within the limits. The control algorithm is set to maintain the desired temperature $T_{\mathrm{d}} = 220$ °C as long as the oil temperature in the tank is within the specified limits. When the oil temperature in the tank drops below the lower limit or exceeds the upper limit, the final temperature target is ignored until the temperature in the tank returns to within limits, with hysteresis width of 5 °C.

It can be seen that there is a significant drop in temperature towards the consumers between 21st and 22nd hour (Fig.3(d)) but also, more importantly, a significant drop below minimum thermal oil operating temperature $T_{\mathrm{min}} = 120$ °C (Fig.3(f)). On the other hand, predictive control had prepared for this critical moment, when the combined available power from the utilization unit and the heater was insufficient to meet current demand, by proactively increasing the oil temperature in the tank, drawing more power from the heater than was needed at that time. So, during the period preceding the critical moment, more power was taken from the heater than necessary to handle the critical phase effectively. The same thing happened between the 14th and 16th hour. It can also be observed that at the start of the day, specifically from the 2nd hour, when excess heat is available from the utilization unit, it is used to raise the oil temperature in the tank close to its maximum level.

Moreover, in this day shown, the initial temperature of the oil in the tank in the case of classic control was set at $T_{\mathrm{in}}(0) = 150$ °C, and the day ended with $T_{\mathrm{in}}(N) = 133$ °C, which means that the next day will start with even less favorable initial conditions and it will be more prone to constraints violation, while predictive control ensured the repeatability condition.

VI. CONCLUSION

This paper introduces a method for determining the operation schedule of the oil heating process in natural gas fractionation, in which a cogeneration waste heat utilization unit and a gas heater are used. The model, control criteria and technical constraints are presented, which were translated into a linear program that determines the optimal actuation of the utilization unit and the gas heater. An on-line control strategy that uses this optimal schedule is also developed. The results on the case study are presented, which show the economical and robust operation of the system in the proposed

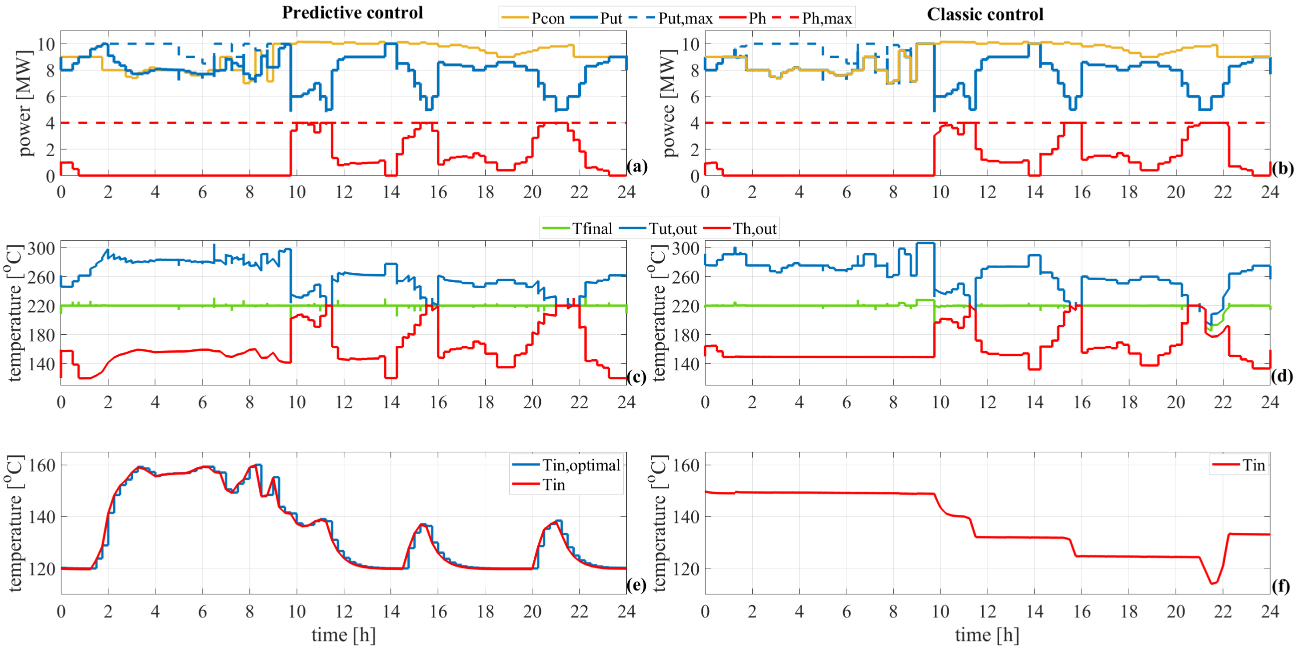

Fig. 3. Simulation results during one day. Maximum and realized power of heater and utilization unit and thermal load in case of predictive control (a), and classic control (b). Simulation of the oil temperature at section outlets and final oil temperature in case of predictive control (c), and classic control (d). Simulation of oil temperature in the tank during the day in case of predictive control (e) and classic control (f).

way with significantly improved performance compared to classic control.

REFERENCES

[1] B. Looney. (2022) Full Report—BP Statistical Review of World Energy 2022.

[2] E. Gallestey, A. Stothert, D. Castagnoli, G. Ferrari-Trecate, and M. Morari, "Using Model Predictive Control and Hybrid Systems for Optimal Scheduling of Industrial Processes," *at - Automatisierungstechnik*, vol. 51, no. 6, pp. 285–294, 2003.

[3] D. Mayne, J. Rawlings, C. Rao, and P. Scokaert, "Constrained model predictive control: Stability and optimality," *Automatica*, vol. 36, no. 6, pp. 789–814, 2000.

[4] K. Kruger, M. Rode, and R. Franke, "Optimal control for fast boiler start-up based on a nonlinear model and considering the thermal stress on thick-walled components," in *Proceedings of the 2001 IEEE International Conference on Control Applications (CCA'01)*, 2001, pp. 570–576.

[5] F. Rukavina and M. Vašak, "Mixed-integer Modelling and Optimization of a Heat Source and a Storage System," *IFAC-PapersOnLine*, vol. 55, no. 20, pp. 133–138, 2022, 10th Vienna International Conference on Mathematical Modelling MATHMOD 2022.

[6] M. H. Marzaki, M. H. A. Jalil, H. M. Shariff, R. Adnan, and M. H. F. Rahiman, "Comparative study of Model Predictive Controller (MPC) and PID Controller on regulation temperature for SSISD plant," in *2014 IEEE 5th Control and System Graduate Research Colloquium*, 2014, pp. 136–140.

[7] N. Guo, Z. Yu, L. Zhao, M. Li, J. Yang, J. Tang, Q. Li, and D. Li, "Research and application of waste heat recovery and cascade utilization for low-grade flue gas in industrial field," in *2019 IEEE Innovative Smart Grid Technologies - Asia (ISGT Asia)*, 2019, pp. 630–635.

[8] M. Tahmasebzadehbaie and H. Sayyaadi, "Techno-economic-reliability assessment of a combined NGL refinery and CCHP system driven by wasted energy of flare and flue gases," *Process Safety and Environmental Protection*, vol. 171, pp. 152–166, 2023.

[9] S. Mokhatab and W. A. Poe, "Chapter 10 - Natural Gas Liquids Recovery," in *Handbook of Natural Gas Transmission and Processing*, 2nd ed., S. Mokhatab and W. A. Poe, Eds. Boston: Gulf Professional Publishing, 2012, pp. 353–391.

[10] The MathWorks Inc., "MATLAB version: 9.9.0.2037887 (R2020b)," 2020.

[11] IBM Corp., "IBM ILOG CPLEX version 12.10," 2019.